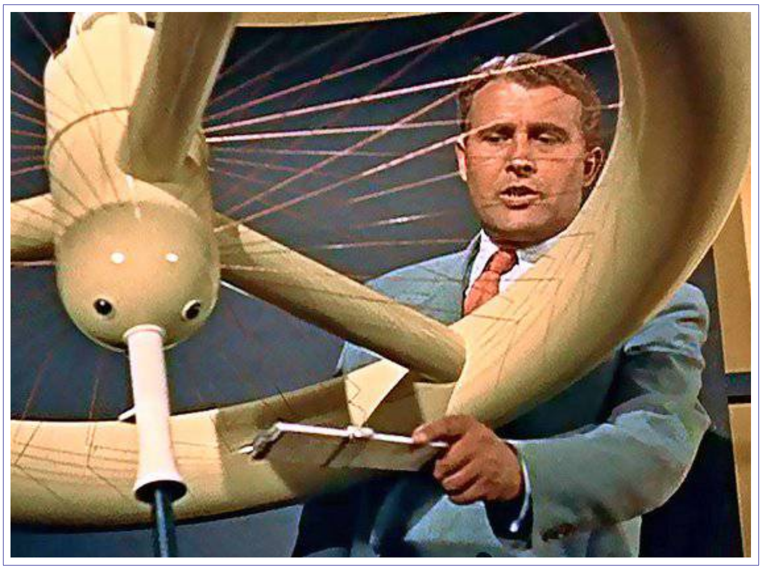
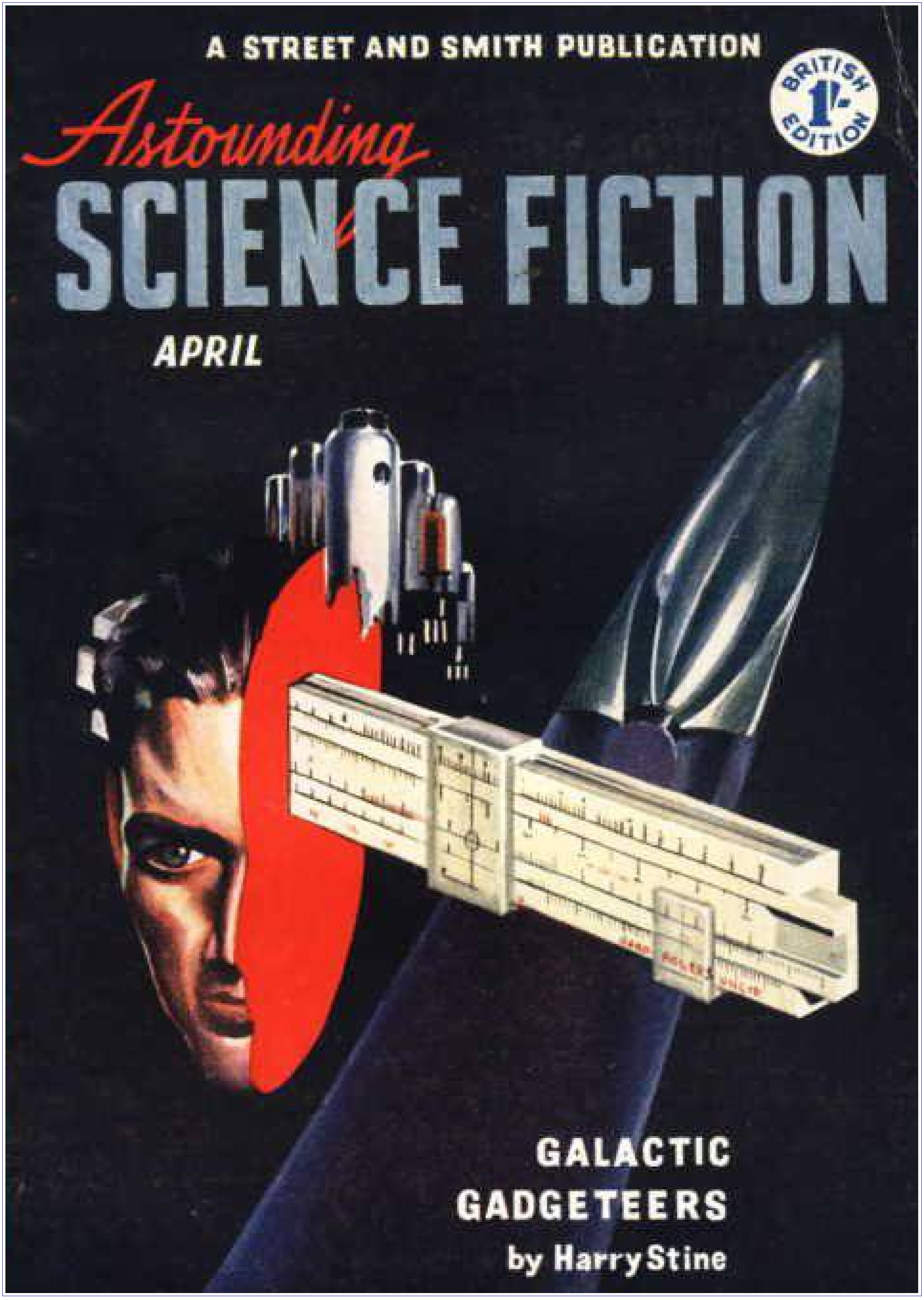
The slide rule: an analog computer



Von Braun with his slide rule (Courtesy of Disney Studios)

"Houston, Tranquility Base here. The Eagle has landed"

with these words Neil Armstrong announced the landing on the moon. One of the on-board computers was a pocket slide rule, supplied to all the Apollo missions. Invented in 1622 this tool came in space: an history long time forgotten, overtaken by a digital age that seems to exist forever.



The analog calculators were considered irreplaceable and their use was imagined also far into the future: what will be tomorrow of our computers?

Logarithms and Gunter's Scale

The mathematician John Napier argued: *"Perform calculations are slow and often difficult task and the boredom that ensues is the main cause of the alienation that people feel towards mathematics"*. He found a solution in 1614 with the discovery of logarithms, published in *"Mirifici logarithmorum canonis descriptio"*, capable of expressing any positive number via powers. Since the product of two powers with the same base is a power with the same base and exponent given by the sum of the exponents, with logarithms multiplications and divisions can be made as simple additions and subtractions.

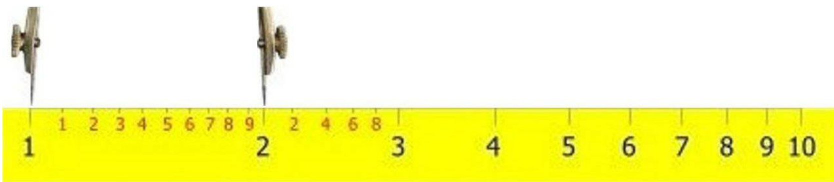
In 1617 Henry Briggs rationalized the logarithms printing them in form of tables, which allowed to count up to fourteen decimal places, just as a modern calculator! To multiply two numbers just look out for their logarithms and add them together: the result is the number whose logarithm correspond to the sum. In practice the logarithm of a number in a certain base is the exponent to which the base must be raised to obtain the number. The logarithm of 10,000 in base 10 is 4 (10^4 = 10,000) and 10,000 x 1,000 become $10^4 x 10^3 = 10^{4+3} = 10^7$ = 10,000,000. Multiplication and division of exponents allow to find squares, cubes and roots. Things get complicated when dealing numbers other than 10: we need a volume with more than a million values, but the tables had a very long life as they were cheap and their precision made them indispensable for astronomers and navigators. They were constantly improved and publications ceased around 1975. Today the logarithms are used mainly to solve exponential equations.

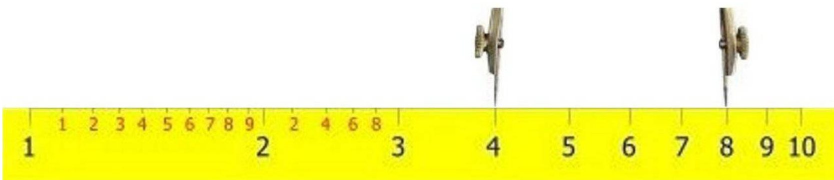
With logarithms it is impossible to work quickly as the consultation of the tables is very laborious and in 1620 Edmund Gunter, to expedite the proceedings, designed the logarithmic scale by placing numbers on a ruler at a distance from the origin proportional to the value of their logarithm. Here is the table:

1	2	3	4	5	6	7	8	9	10
0	0,301	0,477	0,602	0,699	0,778	0,845	0,903	0,954	1

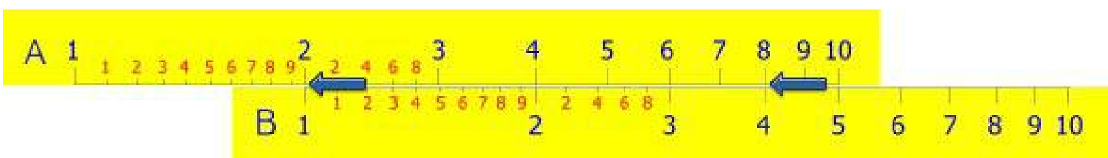
Now we can try to construct the scale: the 1 is the starting point, the 2 is located at 3.01 cm, the 3 at 4.77 and so on up to 10. We can therefore represent each number as we can read, for example, the number 3 as 30, 300, 3,000, 0.003, 0.3, etc.

How far it is possible, for reasons of space, we now add the minor divisions (logs between 1 and 99) and instead of search the logarithms in the tables we can simply add them with the help of a compass. To perform 2 x 4 we open the compass between 1 and 2 and then, keeping the same aperture, we put a tip on 4: the other tip will indicate the result and to divide we use the opposite proceeding.





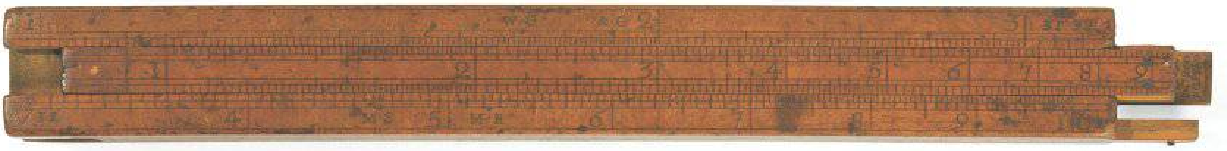
The Gunter's Scale remained in use for 300 years despite the slide rule was invented in 1622. In that year William Oughtred marked the logarithmic scales on two sliding parallel rulers: an innovation that allows the direct reading of the result.



2 x 4 with the Oughtred sliding rulers

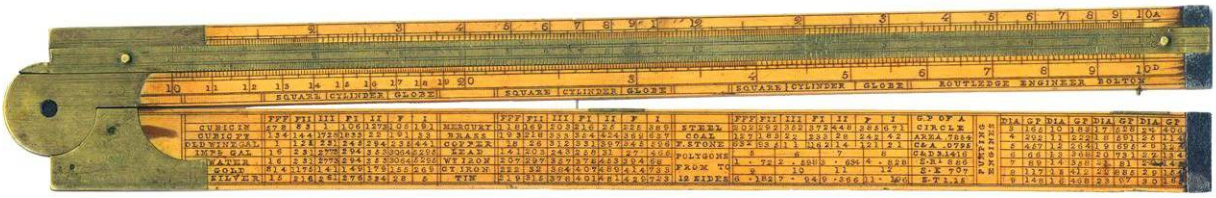
The slide rule

In 1654, just few years after the invention of Oughtred, Robert Bissaker made the *"Gauging Rule"*, with 4 slides, specialized in measuring the contents of the barrels of wine, beer or spirits and calculate the tax burden. A very succesfull instrument that was marketed for over 300 years.



The Gauging Rule of Thomas Everard, half of the eighteenth century

In 1677 Henry Coggeshall created the *"Carpenter's Slide Rule"*, mounted on two wooden rulers with the gradation in inches, the central sliding scale in bronze and several other scales to solve various problems. It is a combined instrument that has allowed the common people to measure and calculate, remained in use until the beginning of 1900 especially in shipyards and workshops.

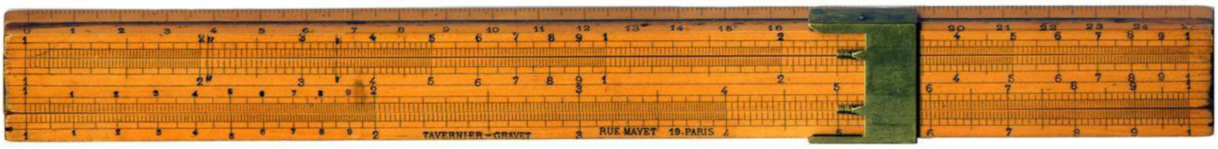


Carpenter's Slide Rule, ca. 1840

At the beginning of 1700 there were slide rules specific to all the needs of the time: the Carpenter's Slide Rule was used to find the volume and weight of shipments of timber, the Gauging Rule to calculate the taxation of beer barrel, while the Gunter's Scale allowed a great work: the mapping of the United States.

Towards mid-800, however, there was a pressing need for computational tools not only specialized in tax or workshop use and, essentials for the design of steam engines and the development of railroads, generic slide rules began to appears, soon became the secret weapons of the industrial revolution.

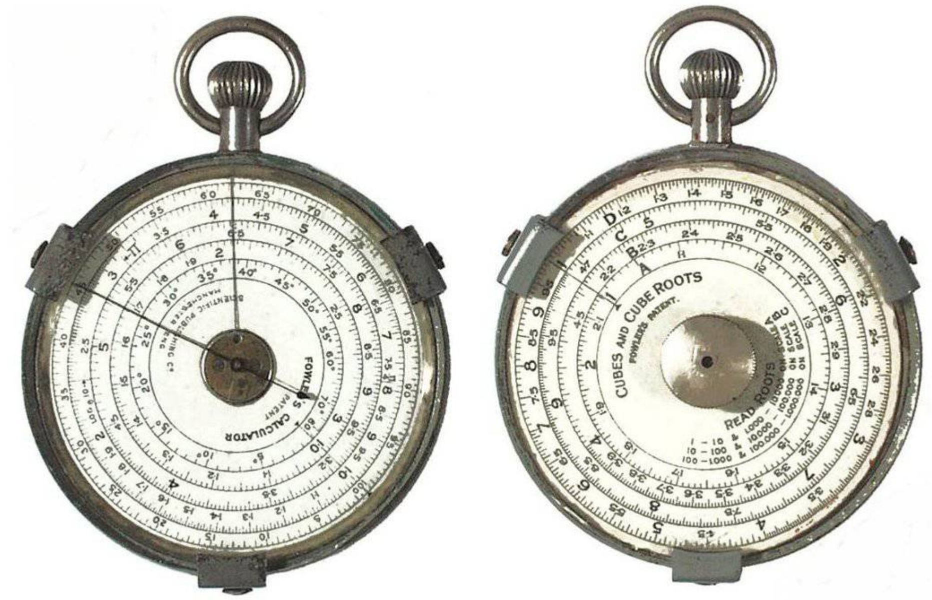
In 1859 the French artillery lieutenant Amédée Mannheim perfected the scales introducing the movable cursor: the modern slide rule was born.



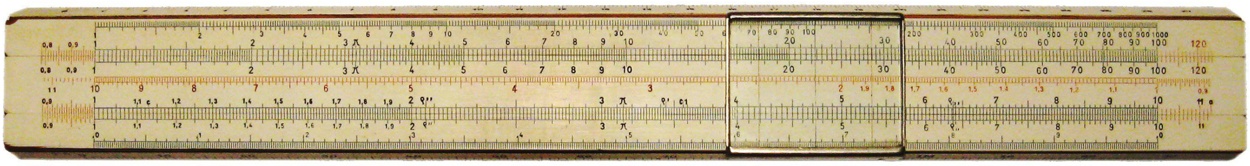
With the Mannheim slide rule appears the cursor, ca. 1860

Around 1920 the slide rule had assumed its final form: Einstein used it to develop the theory of relativity, Marconi for the radio, Fermi for the atomic bomb, Korolev for the Sputnik program and Von Braun for the engines of the Saturn V, the Apollo vector. In order to improve accuracy, proportional to the length of the scales, where produced very large models also circular or cylindrical.

Fuller's cylindrical slide rule, ca. 1915

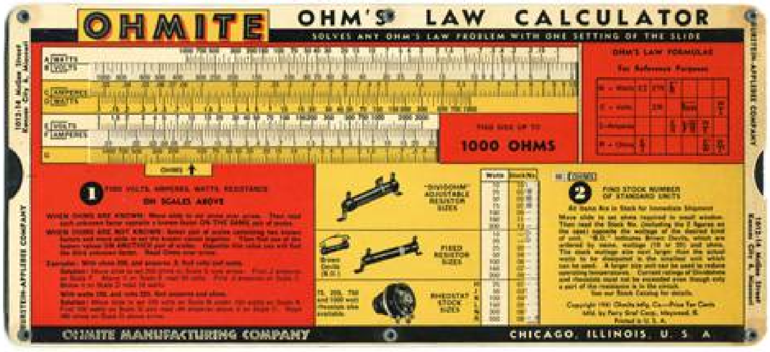


Fowler's round pocket slide rule, front and rear, ca. 1920



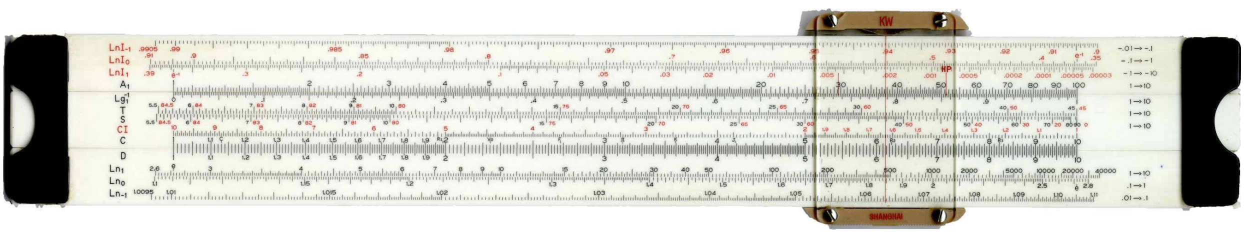
Classic Nestler slide rule, the model preferred by Einstein, ca. 1930

The first computers appeared around 1946, but they were huge and expensive, the same IBM planned to sell up to four a year, and the slide rules seemed irreplaceable. Nobody imagined a world without them: they served to housewives in the kitchen, to tracing the routes on the ship *"Star Trek"*, appeared on the cover of Playboy, were also proposed in the form of cufflinks and tie clips.



Slide rule tie clip and Ohm's law calculator, ca. 1955

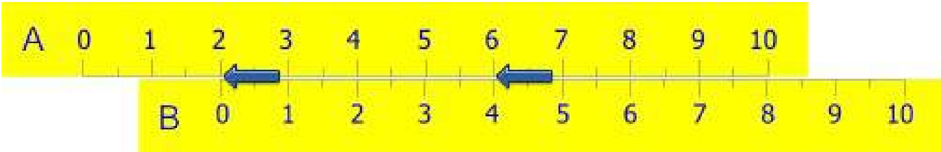
Walt Disney had a simplified model for the children, was built in Braille for the blind, with scales dedicated to solving statistical problems and also in hexadecimal, octal or binary for computer programmers: it was the laptop of the era, always sticking out of engineers' pocket. A true sign to identify the category.



Modern slide rule, ca. 1960

The basis of the slide rule

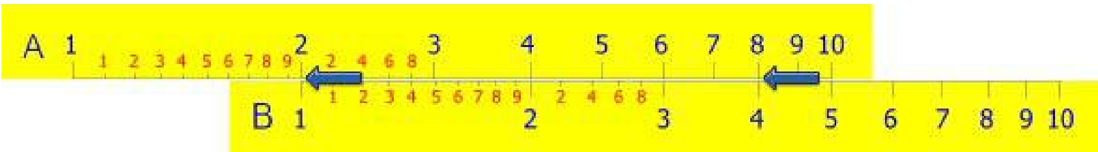
The slide rule, being an analog instrument, replace the mathematical functions with linear measurements. To show how it works let's start to see how we can execute an addition using two common metric rules: to add 2 and 4, align first the 0 of the rule B with the 2 of the rule A. We have set 2+ and the sum can be read on the mark of slide A corresponding to the second addendum.



To perform 2 + 6 we don't need to move again the rule (set on 2+), but just read the sum directly on the figure 6 of the B rule. To subtract we use the opposite proceeding.

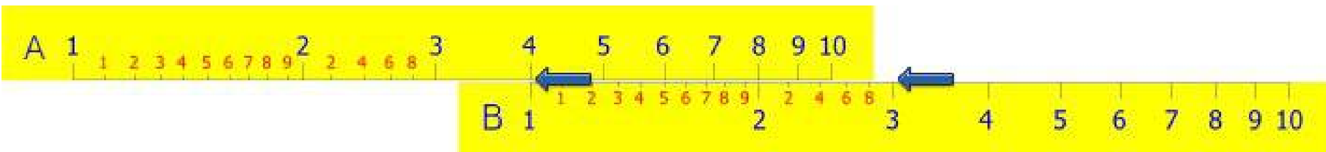
From the accuracy of the construction depends the precision of the results but, also dividing further the scales, it is not possible to operate with numbers greater than 100. It 'is therefore clear that, as regards the addition and subtraction, the slide rule is much less practical than the abacus and to any other type of calculator. This system, however, becomes very powerful if the scales are drawn using the logarithmic succession that we have seen previously.

To perform 2 x 4 we align the 1 of scale B in correspondence of 2 in scale A and the result can be read on the same scale above the 4 of scale B.

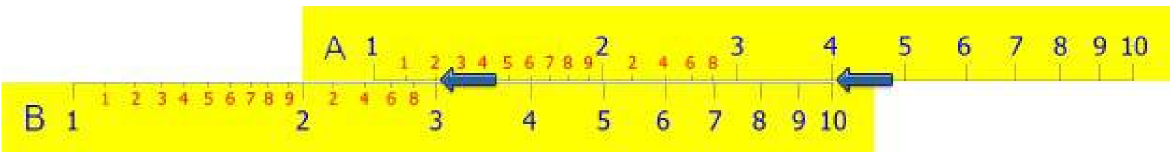


We now have a tool that can perform multiplication (2x with this setting); the previous picture also shows how to perform 8/4: just put the 4 of scale B under the 8 of scale A and read the result on the same scale above the 1 in scale B.

There are also some disadvantages: if we want process 4x3 the slides are positioned as follows:



The total is now located out of the scale. To solve this problem, we need to use the 10 of the rule B, instead of the previous 1:



So we obtain 1.2, but the right total is 12: the slide rule gives only the numbers and how to locate the dot or how to add tens or hundreds we must find by ourselves.

$$\frac{\sqrt{\sqrt{\sqrt{(0.424 \times 6.13)^3}}}}{2.63 \times 0.41 \times 3.27} \times \frac{\left[\frac{0.008}{21 \times 63}\right]^3}{0.000000278}$$

This was just a brief outlook on how the system work, but the slide rule has many other scales and can reach the computing power of a modern calculator. His only flaw is the poor readability, but an equation such as this on the left can be solved in few minutes. The secret is: practice and to start you can download from my website several paper templates of slide rules very easy to build.

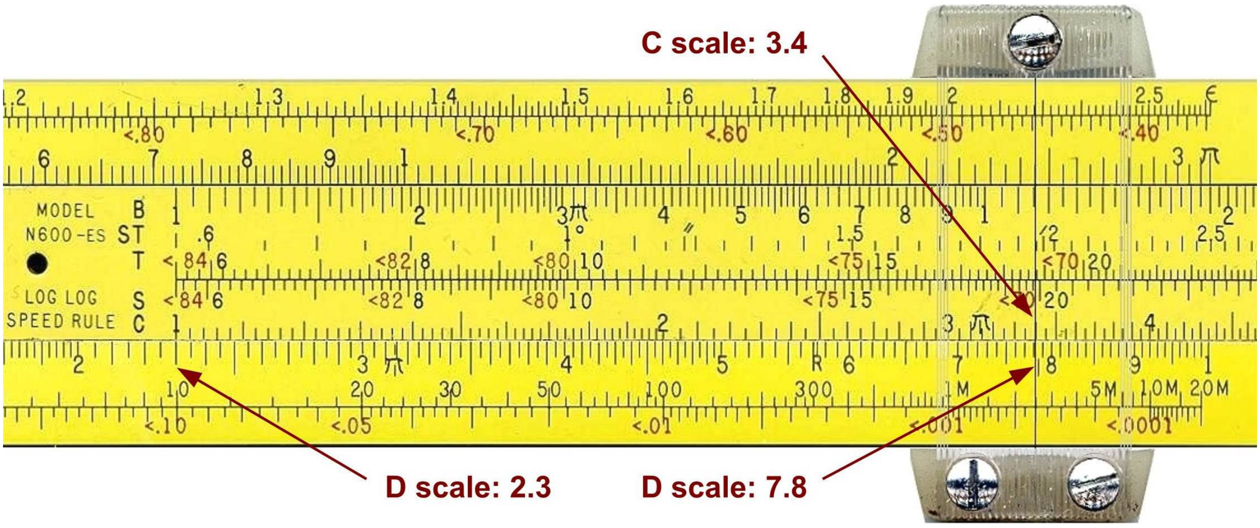
Examples to practice

In the modern slide rule the scales are indicated by letters: the two most important, equivalents to the Ougtred A and B, are on the slide (C) and on the body (D). The others are used to simplify the calculations when you are in the presence of square roots (A and B), cubes and cube roots (K), exponential (LL), sine and tangent (ST and T), multiplication and division for pi (CI and CF), tangent (T), etc. up to more than 30.

Multiplication (uses C and D scales)

Example: 2.3 x 3.4

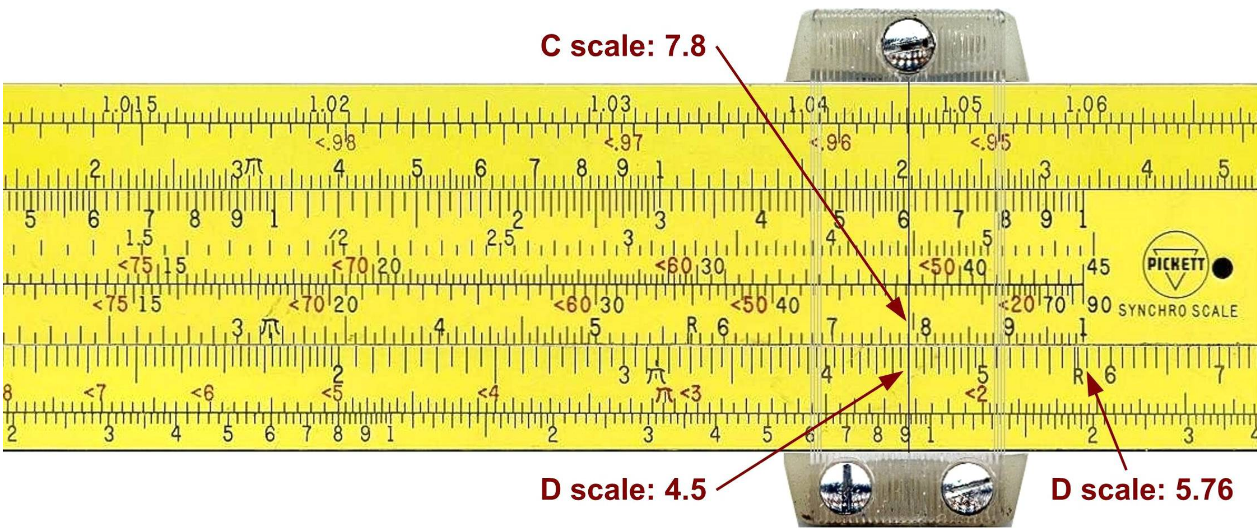
- slide the leftmost '1' on **C** over 2.3 on the D scale;
- move the cursor to 3.4 on the **C** scale;
- the cursor is on the **D** scale just a bit over 7.8. The correct answer is 7.82.



Division (uses C and D scales)

Example: 4.5 / 7.8

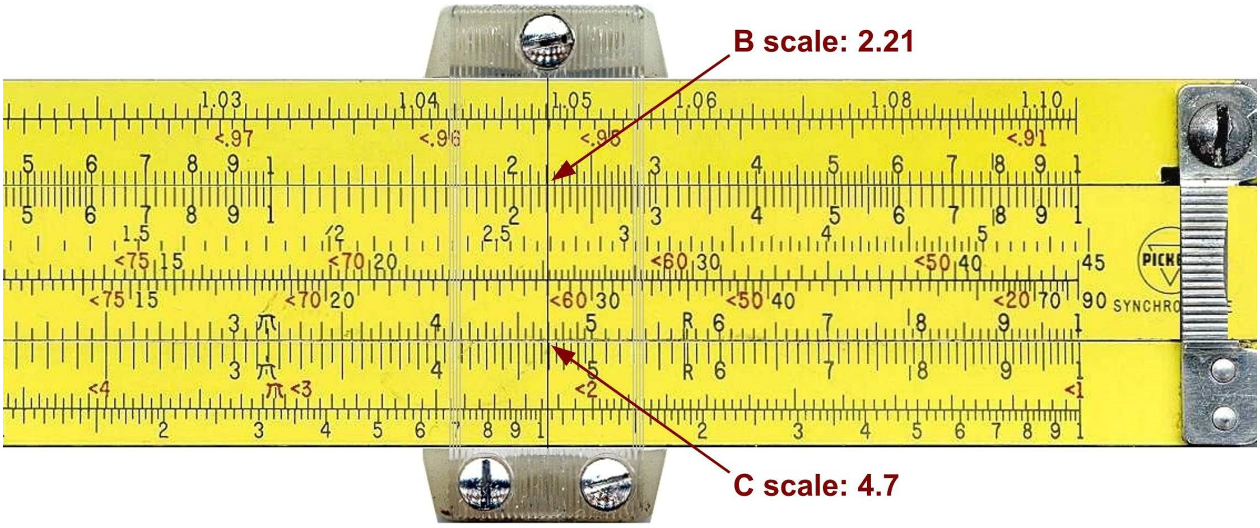
- move the cursor to 4.5 on the **D scale**;
- slide 7.8 on the **C scale** to the cursor;
- the rightmost '1' is now at 5.76 on the **D scale**. We know that the result of 4/8 is near 0.5, so we adjust the decimal place to get 0.576. The correct answer is 0.576.



Squares (uses C and B scales)

Example: 4.7^2

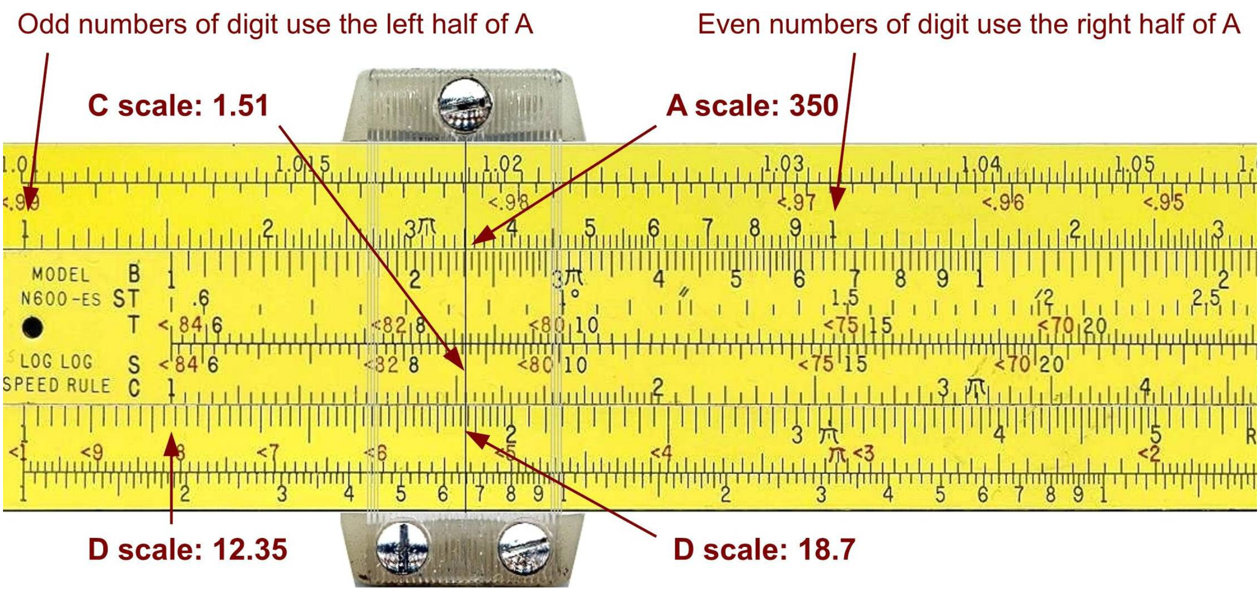
- move the cursor to 4.7 on the **C scale**;
- the cursor is now at 2.2 on the **B scale**;
- we know that the result of 5^2 is near 25, so we adjust the decimal place to get 22. The correct answer is 22.



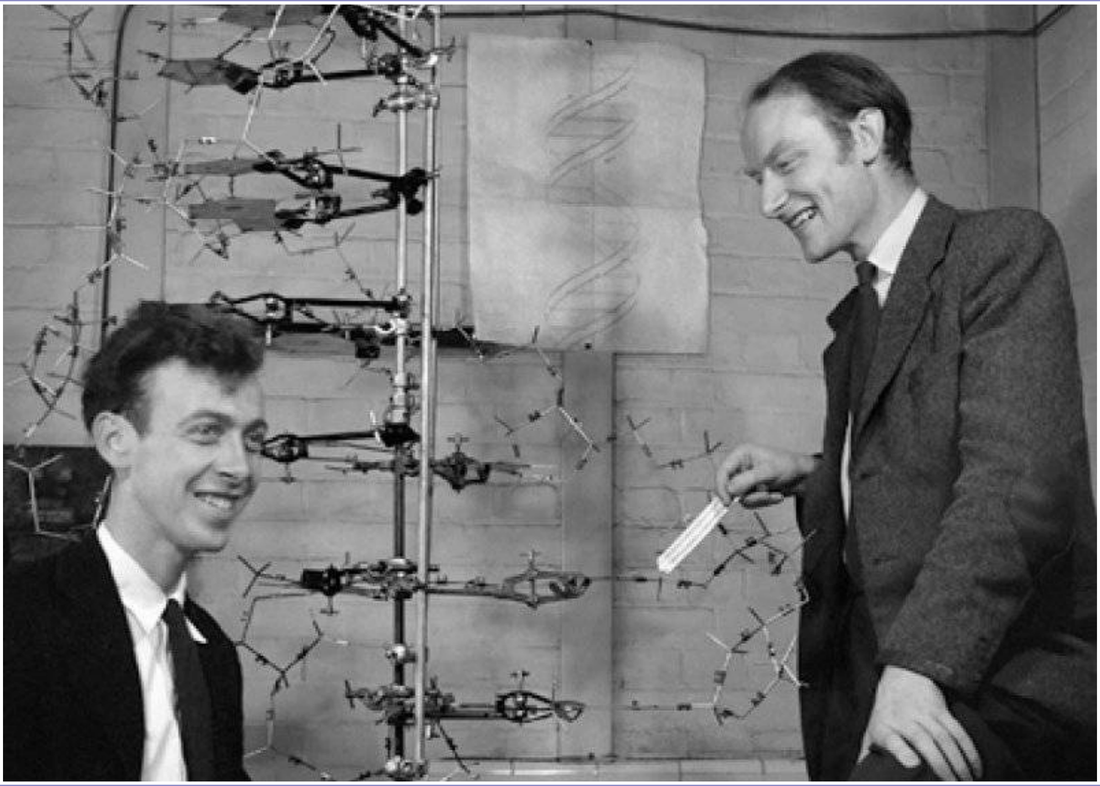
Square roots (uses C and A scales)

Let's do now an example of a division where the numerator is a square root: $\frac{\sqrt{350}}{1.51}$

- the **A scale** has two similar halves. The left half is used to find the square root of numbers with odd numbers of digits; the right half is used for numbers with even numbers of digits. Since 350 has an odd number of digits we'll use the right half of the scale.
- moving the cursor over the 350 of the **A scale** we get its square root, 18.7, on the **D scale**;
- now we match the number 18.7 with 1.51 of the **C scale**: on the D scale, in correspondence with the leftmost index '1' on **C scale**, we read the answer: ca. 12.35.



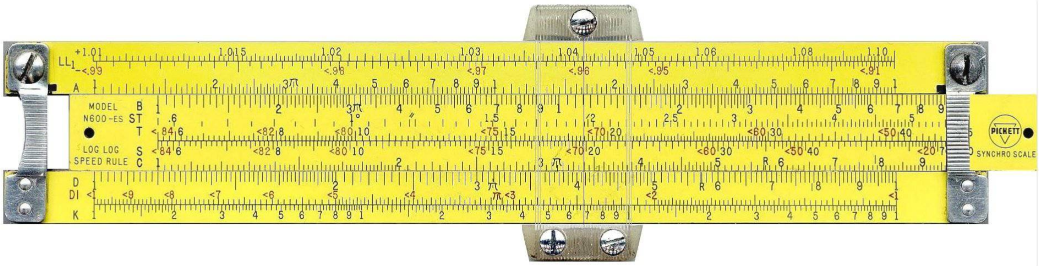
A calculator would have been just a little more precise, finding 12.3896. This slight approximation has not prevented Von Braun to design space stations and send men on the moon: the slide rule is in fact less difficult than it sounds and the secret is just to be accurate and to practice.



Francis Crick and James Watson, discoverer of DNA, with a pocket slide rule. 1953

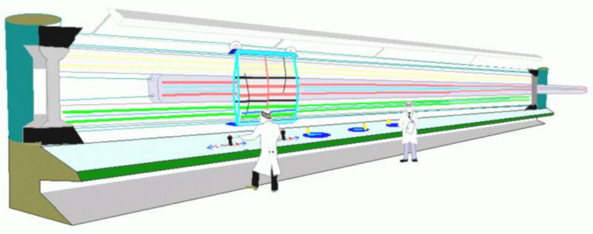
The world is changing: the computer arrives in Vatican, 1960

The twilight of the analog era



The Pickett ES 600 supplied to the Apollo missions

In 1969 the slide rule was landed on the moon on-board the Apollo 11: a very long career which began more than 350 years before. However these tools are only accurate to three decimal places and the engineers had to make continuous estimates with the help of their experience. Approximating the calculations for excess created the myth of the *"Olde Good Things"*, but the modern structural analysis required now exact results, thus promoting the development of small electronic calculators. These were of course designed



using slide rules: Robert Ragen said to have literally consumed two to realize in 1963 its revolutionary *"Friden 130"*. It was still too cumbersome and increasing complex slide rules continued to be built: in the Soviet Union it was constructed one, electromechanical, of 14 meters in length. Made in the Kalashnikov's workshops was baptized with the name of the biblical monster Behemoth, really appropriate for such a giant.

Finally in 1972 the Helwett Packard, advertising it as *"Innovative electronic slide rule"*, put on sale the first economic scientific calculator, 50 times smaller than the competitors and so modern that it is still on the market. The capabilities of the new HP 35 were indispensable, Forbes cites it among the 20 objects that have changed the world, and analog computers disappeared from the market in a flash.

The new HP-35, so called "the slide rule killer"

Shortly before 1972 the president of a very old slide rules factory had declared:

"No matter what aspect of the future we consider, we can see a continuing and important role for the company that supplies the engineering profession with the tools it needs. We have been that company for over 100 years, and we intend to be that company for the next 100 years".

Never somebody were less prophetic: only two years after his company closed and the slide rules, produced over the centuries in more than 60 million units, came out of the story. But our *"hero"*, reliable and environmentally friendly, it is always necessary to pilots and military and perhaps the adventure is not over yet, as in the Asimov' science fiction novel *"The feeling of power"* that, assuming a return to the old methods of calculation, ends with these words: *"Nine times seven, thought Shuman with deep satisfaction, is sixty-three, and I don't need a computer to tell me so. The computer is in my own head. And it was amazing the feeling of power that gave him"*.

The first computers were bulky and not very powerful, but they were able to replace 150 engineers equipped with slide rule: their time had come

The slide rule, invented in 1622, went to the moon